A New Edge Detection Operator Based on Modified Tsallis Entropy

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***Abstract*-Edge detection is an important first step operation of obtaining information from an image. In this paper, a new edge detection operator is proposed, which is based on the principle of evaluating the local fluctuation of pixel values. The pixel values within a mask are normalized so that they can be treated as probabilities. Then, modified Tsallis entropy of the probabilities is calculated. Edges are detected where the value of this entropy is below a properly chosen threshold. The obtained edges are further refined by a morphological operation. Generalizations to color images are also considered. The new operator shows promising effect in the simulation.**

***Keywords*-Image processing; edge detection; operator; Tsallis entropy; color image**

I. INTRODUCTION

Information obtained from the visual channel is a major source of our knowledge of the world. The past one or two decades have witnessed the quick development and popularization of digital image acquisition devices such as scanners and digital cameras. The computing power of personal computers has been steadily improved at tremendous speed. With the pervasive World Wide Web, sharing of images and videos has never been as easy as it is today. Pushed by these impetuses, digital image processing [1] has now become indispensable, and has gained more attention of research in the realm of information science and management systems.

In both human and machine vision systems, edges in images are of crucial significance in understanding the contents of images. Edges provide fundamental information of the contour of an object, which helps a human or a machine to recognize the object. Edge detection has therefore become an important task in image processing. Intuitively speaking, edges are usually accompanied by apparent differences in pixel values. Such differences can be detected by using the first or second order derivatives of the pixel values. The maximum value points of the first order derivative or the zero-crossing points of the second order derivative are most likely to be edges. A so-called edge detection operator is usually defined by calculating a derivative. The Roberts operator, the Sobel operator, and the Prewitt operator are based on the first order derivatives [1]. The Laplacian operator is a second order derivative based edge detection operator [1]. It is sensitive to noise, which can be remedied by firstly smoothing the image and then applying the second order derivative edge detection operator, as is done in the LoG (Laplacian of Gaussian) operator [2]. There are also edge detection operators that are not directly based on derivatives. For example, the Canny operator is an optimized operator derived under certain conditions [3].

The above mentioned methods are some of the most commonly used classical operators for detecting edges in images. Besides, recent efforts have produced many novel approaches for edge detection [2], [4]–[10], some of which are summarized as follows. Based on the Laplacian operator, Wang [2] proposed the optimal edge-matching filter-based edge detector (OED) and the multistage median filter-based edge detector (MED). Laligant and Truchetet [4] proposed an edge detection approach based on the nonlinear combination of two polarized derivatives. Agaian et al. [5] introduced the Boolean function derivatives and applied them to identifying edges in binary and grayscale images. Evans and Liu [6] proposed an efficient color edge detector based on vector differences. Using the entropy of brightness in a local region of a picture, Shiozaki [7] proposed the entropy edge detection operators for monochrome and color pictures. Based on the least squares support vector machine (LS-SVM) with Gaussian radial basis function kernel, Zheng et al. [8] obtained a set of gradient operators and the corresponding second derivative operators. Yuksel [9] presented a neuro-fuzzy (NF) operator for edge detection in digital images corrupted by impulse noise, which is constructed by combining a desired number of NF subdetectors with a postprocessor. Basturk and Gunay [10] presented a cellular neural network (CNN) based edge detector optimized by differential evolution (DE) algorithm.

The Tsallis entropy is a generalization of the standard Boltzmann-Gibbs-Shannon entropy, which was proposed by Tsallis in 1988 [11]. In recent years, the Tsallis entropy have found quite a few applications in image processing such as image thresholding [12], segmentation [13], and edge detection [14]. Because it contains an adjustable parameter q, the Tsallis entropy is considered to be more flexible than the usual Shannon entropy when being used in image processing [14].

The motivation of this paper is to propose an easily computable new operator for edge detection. This new operator is based on the principle of evaluating the local fluctuation of the pixel values. First, the pixel values within a mask are normalized so that they can be treated as probabilities as was done in [7]. Then, a modified Tsallis entropy of the probabilities is calculated. Edges are detected where the value of this entropy is below a properly chosen threshold. The obtained edges are further refined by using a

morphological operation. In the simulation, the proposed operator shows promising effect compared with some well-known existing operators.

The contribution of this paper can be seen by comparison with some other edge detection methods based on Tsallis entropy. For example, in [14], the Tsallis entropy is used for choosing the threshold of edge intensity, while the edge intensity is calculated from some gradient operators. In this paper, however, the gradient operators are not used. Instead, a modified Tsallis entropy is calculated and used to generate the edge intensity. Therefore, the principle of the new operator is quite different from that in [14].

The structure of the paper is as follows. In Section II, the Tsallis entropy is briefly introduced. In Section III, the new operator for edge detection is presented, where the extension of the method to color images is also shown. Some simulation results are placed in Section IV. The paper is summarized in Section V with conclusions and outlook.

II. Tsallis Entropy

Suppose that we have a set of possible events whose probabilities of occurrence are $p_1, p_2, \ldots, p_n$ such that $\sum_{i=1}^{n} p_i = 1$. Then, the Tsallis entropy is defined as [11]

$$S_q \equiv k \frac{1-\sum_{i=1}^{n} p_i^q}{q-1} \tag{1}$$

where k is a conventional positive constant. It can be proved that the Shannon entropy is the limit of the Tsallis entropy as $q \to 1$, i.e.,

$$H = \lim_{q\to 1} S_q = -k \sum_{i=1}^{n} p_i \ln p_i \, . \tag{2}$$

III. A New Edge Detection Operator

A. Problem Statement

The task of edge detection can be formulated as follows. Suppose that we have an M by K pixels grayscale image whose value at the (w,m) th pixel is denoted by $g(w,m)$, where $(w,m) \in \{1,\ldots,K\} \times \{1,\ldots,M\}$. We try to generate a binary image $b(w,m)$ where the positions of edges and non-edges are marked by 1 and 0, respectively.

B. Tsallis Entropy Operator

Denote the nonnegative pixel values of the (w,m) th pixel and its $n-1$ neighboring pixels by $g_1, g_2, \ldots, g_n$, respectively. In order to apply (1), we normalize the pixel values so that they sum up to 1, as was done in [7]:

$$p_i = \frac{g_i}{\sum_{j=1}^{n} g_j}, \quad i = 1, \ldots, n. \tag{3}$$

Now, $p_1, \ldots, p_n$ can be treated as a set of probabilities, upon which (1) can be applied.

However, to design an operator for edge detection, it is important to reduce the load of computations, since the operator is usually computed repeatedly over the whole image. Now let us see if there is any unnecessary computation in (1). For the sake of simplicity, we only consider the Tsallis entropy with $q > 0$. We can see in (1) that the constant k does not affect the relative magnitude of Tsallis entropy. Therefore it can be removed from the computation. Similarly, once q is selected, the denominator $q-1$ needs not to be computed. Moreover, our approach only depends on the relative greatness of Tsallis entropy and, therefore, the term 1 in the numerator can be removed, too, as it has no effect on the relative greatness. Thus, we obtain the following modified definition of Tsallis entropy:

$$\sigma_q \equiv \begin{cases} \sum_{i=1}^{n} p_i^q & 0 < q < 1, \\ -\sum_{i=1}^{n} p_i^q & q > 1. \end{cases} \tag{4}$$

Next, we shall explain the mechanism of the new operator. Using the concavity of the expression in (4), we can prove the following inequalities regarding the maximum and the minimum of σ_q.

$$1 \le \sigma_q \le n\left(\frac{1}{n}\right)^q \tag{5}$$

for $0 < q < 1$ and

$$-1 \le \sigma_q \le -n\left(\frac{1}{n}\right)^q \tag{6}$$

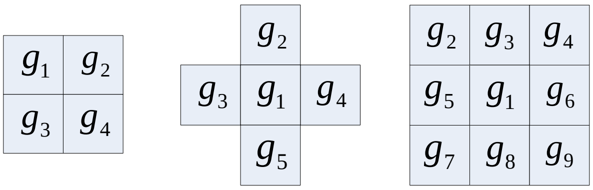
for $q > 1$. Moreover, we can prove that σ_q reaches its maximum when $p_1 = p_2 = \cdots = p_n = 1/n$, namely, when $p_1, \ldots, p_n$ are uniform. We can also prove that σ_q reaches its minimum when there is one 1 and $n-1$ zeros in $p_1, p_2, \ldots, p_n$, namely, when $p_1, \ldots, p_n$ are the most concentrated.

Generally speaking, the pixel values within a small area vary more when an edge crosses this area and vary less when no edge crosses it. Accordingly, the normalized pixel values $p_1, \ldots, p_n$ within this area will be far from uniform when an edge is met and will be nearly uniform when no edge is met. Meanwhile, σ_q is small when $p_1, \ldots, p_n$ are far from uniform and is large when $p_1, \ldots, p_n$ are nearly uniform. Therefore, edges can be detected at the positions where σ_q yields small values.

C. Masking and Postprocessing

When using the Tsallis entropy operator, the pixel values $g_1, g_2, \ldots, g_n$ refer to those within a mask. Fig. 1 shows three shapes of masks. The Roberts operator uses 2×2 masks. One

form of the Laplacian operator uses the center and four neighboring pixels. The Sobel, Prewitt, and another form of Laplacian operators use 3×3 masks. In [7], four neighboring pixels or 3×3 neighborhood was suggested to be used for the entropy operator. The number of pixels in the mask should be large enough to get reliable estimate of the operator and to dispel the effect of noise. On the other hand, too many neighboring pixels may blur the positions of edges. Based on these considerations, we choose a 3×3 mask to be used with the Tsallis entropy operator.



(a) 2×2 mask (b) 4 neighboring pixels (c) 3×3 mask

Fig. 1 Three shapes of masks

After applying the 3×3 mask to each pixel of an image and calculating the Tsallis entropy operator according to (3) and (4), the image of σ_q is obtained. Then, a threshold should be chosen to generate the binary edge image. A suitable choice is that 70–90% pixels have values of σ_q greater than the threshold.

In practice, the Tsallis entropy operator shows consistent performance on various sample images. The only flaw is that the width of the edge curves generated is a little too wide. This can be easily adjusted by applying a morphological thinning operation to the binary edge image.

D. Extension to Color Images

The Tsallis entropy operator described in the above is defined on a monochrome image. Therefore, when detecting edges in a color image, the color image should be first converted into a monochrome image. Suppose that g_R, g_G, and g_B are the red, green, and blue components of a pixel in a color image, resepectively. The graylevel value g of this pixel in the converted monochrome image can be generated as the maximum, the average, or the weighted sum of g_R, g_G, and g_B [1]. In the weighted sum method, the selection of the weights w_R, w_G, and w_B is very important. According to related study, the obtained monochrome image is reasonable when $w_R = 0.299$, $w_G = 0.587$, and $w_B = 0.114$ [1]. Thus, the following formula for converting a color pixel into a monochrome pixel is obtained:

$$g = 0.299 g_R + 0.587 g_G + 0.114 g_B . \tag{7}$$

After converting the color image into a monochrome image, the Tsallis entropy operator for monochrome images developed above can be applied.

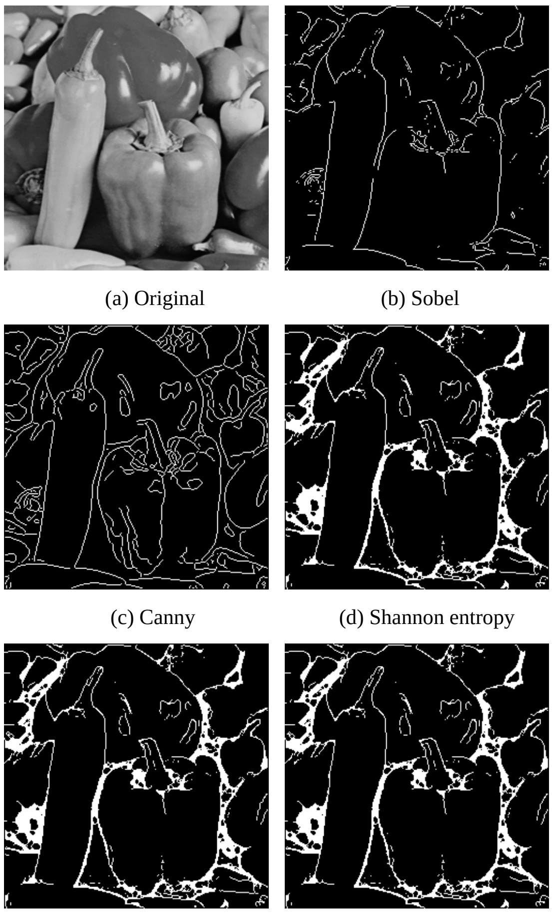
Alternatively, we can first calculate the Tsallis entropy operators $\sigma_q^{(R)}$, $\sigma_q^{(G)}$, and $\sigma_q^{(B)}$ of the three color components of a pixel, respectively, and then combine them in the following way [7]:

$$\sigma_q = k_R \sigma_q^{(R)} + k_G \sigma_q^{(G)} + k_B \sigma_q^{(B)} \tag{8}$$

where

$$\begin{aligned} k_R &= g_R \big/ \left(g_R + g_G + g_B \right) \\ k_G &= g_G \big/ \left(g_R + g_G + g_B \right) \\ k_B &= g_B \big/ \left(g_R + g_G + g_B \right). \end{aligned} \tag{9}$$

In this approach, the three color components are of equal importance. This is better for preserving edge information, because although the blue and red components contribute less to the brightness as implied in (7), they may be equally important as the green component in determining edges.



(a) Original (b) Sobel

(c) Canny (d) Shannon entropy

(e) Tsallis entropy $q = 0.5$ (f) Tsallis entropy $q = 1.5$

Fig. 2 Edge detection of Peppers using existing and new operators

(a) Original (b) Sobel

(c) Canny (d) Shannon entropy

(e) Tsallis entropy $q = 0.5$ (f) Tsallis entropy $q = 1.5$

Fig. 3 Edge detection of House Using Existing and New Operators

(a) Original (b) Shannon entropy

(c) Tsallis entropy $q = 0.5$ (d) Tsallis entropy $q = 1.5$

Fig. 4 Edge detection of color House using Shannon entropy operator and Tsallis entropy operator

TABLE I COMPARISON OF COMPUTATION TIME OF EDGE DETECTION ON A 2.4 GHZ CORE 2 PC (IN SECONDS)

Image	**Method**		
	Shannon Entropy	**Tsallis Entropy** $q = 0.5$	**Tsallis Entropy** $q = 1.5$
Peppers	1.6809	1.3948	1.4425
House	1.7226	1.4431	1.4698
Color House	5.0365	4.2568	4.3848

IV. SIMULATION

The new edge detection operator has been tested on many sample images. Figs. 2 and 3 show the results on the images Peppers and House, respectively. For comparison, the Sobel, Canny, Shannon entropy [7], and Tsallis entropy operators are applied to the images. The Shannon entropy and the Tsallis entropy operators use the 3×3 mask and the thresholds are selected such that the number of edge pixels is 20% of the total number of pixels. The morphological thinning operation in MATLAB Image Processing Toolbox is applied to the binary edge images generated by the Shannon entropy and the Tsallis entropy operators for one time to make the edges look slimmer. Like [14], we select two representative values of q in the new Tsallis entropy operator, $q = 0.5$ and $q = 1.5$.

In Figs. 2 and 3, we can see that the Tsallis entropy operator find edges better than the Sobel operator. Comparing Figs. 3(a) with 3(c)–3(f), we can see that the Shannon entropy and the Tsallis entropy operators detect some edges of bricks at the shadow places of the walls and the chimney that are neglected by the Canny operator. Note that these detected edges of bricks are not noise. We can find them with our eyes by magnifying the original image. They are located at the rather dark regions of the image. This is easy to understand. Since normalization of pixel values to probabilities is somewhat equivalent to amplifying small pixel values and suppressing large pixel values, the Shannon entropy and the Tsallis entropy operators are expected to detect edges better in dark regions.

In Fig. 4, we use the Shannon entropy and the Tsallis entropy operators to detect edges in the color image of House. We first calculate the Shannon or the Tsallis entropy operators of the three color components and then combine them as in (8) and (9). The results are apparently better than those on the monochrome House in Figs. 3(d)–3(f), because the whole contour of the nearer window of the house is detected in Figs. 4(b)–4(d), whereas a segment of a vertical edge of the nearer window of the house is missing in Figs. 3(d)–3(f).

In all the examples, it is interesting to note that the Shannon entropy operator and the Tsallis entropy operator with either $q = 0.5$ or $q = 1.5$ produce nearly identical edge images. This is because that, although they are based on different forms of entropy, their mechanisms are the same.

The Tsallis entropy is usually more efficient to compute than the Shannon entropy, and we have further reduced the load of computation by removing unnecessary operations. In Table I, we can see that the new operator consumes less time in all the runs than the Shannon entropy operator. The size of images in the simulation is 256 by 256 pixels.

V. Conclusions

In this paper, a new operator for edge detection has been proposed. The values of neighboring pixels are normalized to be a set of probabilities. Then, a modified Tsallis entropy of these probabilities is calculated, and the position of edge is determined where this entropy is small. Experiments show that the new operator has promising performance compared with some well-known operators. Especially, the new operator provides similar effect as the Shannon entropy operator with less computation time.

Possible routes of further improvement and future study can be planned as follows. Firstly, in the present approach that we proposed, the threshold for generating the binary edge image is choosing according to experience. In future, an automatic thresholding procedure can be utilized. Secondly, now the Tsallis entropy operator for color images is computed in the RGB color space. In future, we can try to compute the Tsallis entropy operator in the YCbCr color space or other color spaces. Thirdly, although the principles are different, it is still interesting to compare our approach with other edge detection methods based on Tsallis entropy such as that in [14] in the future.

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